SECTION 5

ANALYSIS OF CONTINUOUS SPANS

DEVELOPED BY THE PTI EDC-130 EDUCATION COMMITTEE
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NOTE: MOMENT DIAGRAM CONVENTION

- In PT design, it is preferable to draw moment diagrams to the tensile face of the concrete section. The tensile face indicates what portion of the beam requires reinforcing for strength.
- When moment is drawn on the tension side, the diagram matches the general drape of the tendons. The tendons change their vertical location in the beam to follow the tensile moment diagram. Strands are at the top of the beam over the support and near the bottom at mid span.
- For convenience, the following slides contain moment diagrams drawn on both the tensile and compressive face, denoted by (T) and (C), in the lower left hand corner. Please delete the slides to suit the presenter's convention.

ANALYSIS PROCEDURE – 2 SPAN BEAM

1. Calculate applied loading – self weight, dead, live, etc.;
2. Determine beam section properties and materials;
3. Calculate balanced forces in each span;
4. Calculate net load on beam;
5. Determine support moments;
6. Determine midspan moments;
7. Calculate flexural stresses at support and midspan;
8. Calculate secondary moments.

TYPICAL LONG SPAN PARKING STRUCTURE FRAMING

- Two bay parking structure – 120 feet x 300 feet
- 5” post-tensioned slab spanning between beams
- 16” x 35” post-tensioned beams at 18’-0” on center spanning 60’-0”
- 24” x 35” post-tensioned girders at turnaround
- 24” square columns – typical interior and exterior
- 24” x 30” columns at girders
- All concrete has an 28 day f’c of 5,000 psi

TYPICAL LONG SPAN PARKING STRUCTURE BEAM

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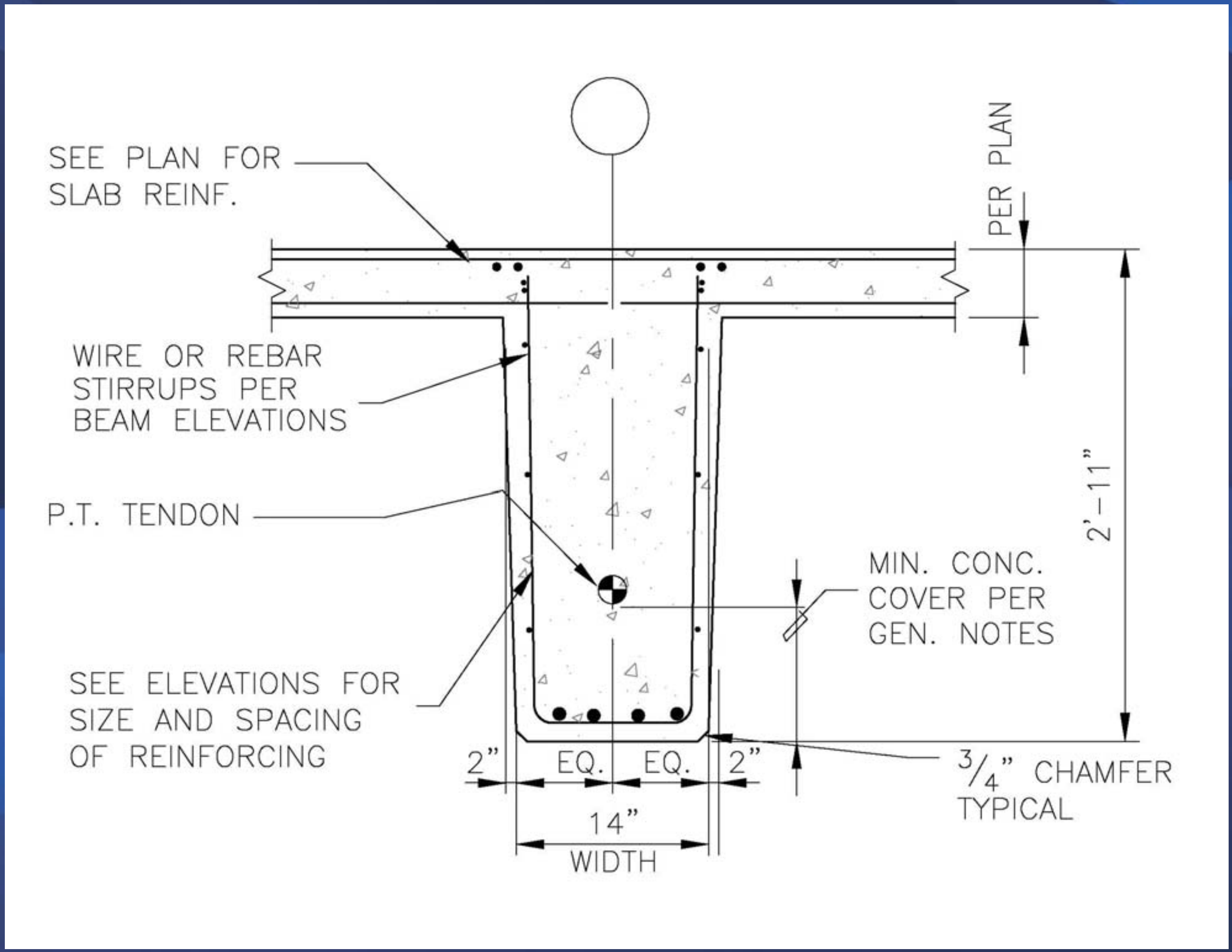
TYPICAL LONG SPAN PARKING STRUCTURE BEAM



LOADING

Dead Load:

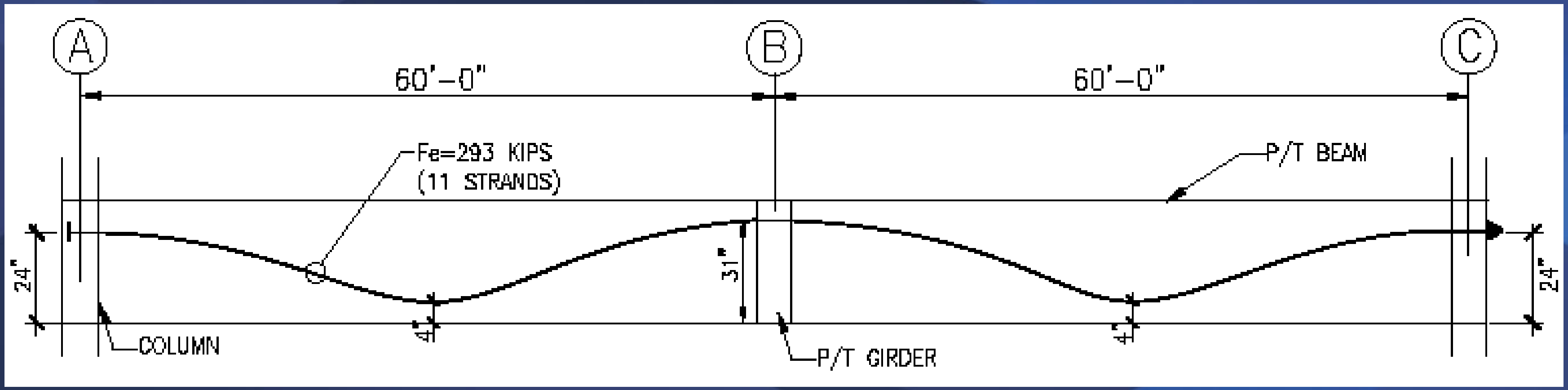
5" P/T slab	63 PSF
Mech'l / elec'l / misc.	5 PSF
P/T beams @ 18 feet on center	28 PSF
P/T girders	3 PSF
Spandrels	5 PSF
Columns	10 PSF
Shear walls	25 PSF

Live Load:

Passenger vehicles only	40 PSF

(Unreducible for slabs / beams / girders)

TYPICAL TWO SPAN BEAM



The beam elevation above is what is typically used in design offices to identify the number of strands and their location along the beam.

The tendon profile shown is what is typically seen in the field. The curvature of the tendons will reverse near the girders and the exterior columns. To simplify the math, a simple parabola will be assumed between the columns and the girder at grid B.

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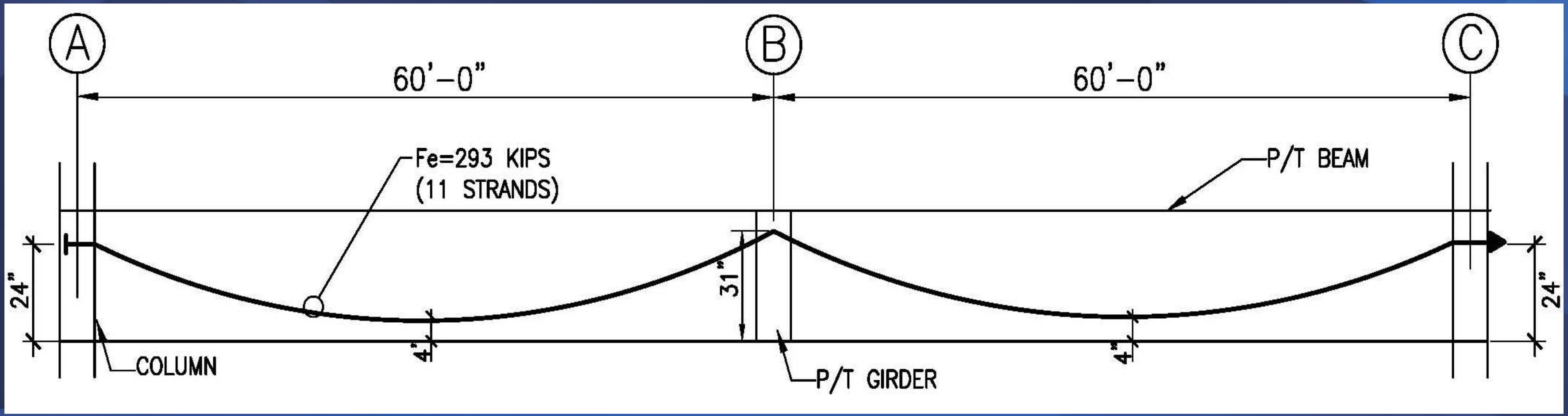
TYPICAL TWO SPAN BEAM W/ SIMPLE PARABOLIC PROFILE



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DEAD AND LIVE LOAD

W_{DL}= 0.096 ksf * 18' = 1.73 kips/foot

W_{LL}= 0.040 ksf * 18' = <u>0.72 kips/foot</u>

W_{TL} = ***2.45 kips/foot***

T-SECTION PROPERTIES – ACI 8.12.2

B_{eff} : Width of slab effective for beam design/analysis

Lesser of: 1) L (beam span) /4 = 60'*12 / 4 = 180"

2) 16*t + b_w = (16*5")+16" = 96" ***(Controls)***

3) One half the clear distance to the next web = (18'*12) – 16" = 200"

t = Slab thickness in inches

b_w = Beam width – For simplicity we will use 16 inches for the full depth of the beam

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T-SECTION BEAM SECTION PROPERTIES

For simplicity, the beam is assumed to be a constant 16” wide

A = (96”*5”) + (30”*16”) = **960 in^2**

CG_t = ((96”*5”*2.5”) + (30”*16”*20”)) / 960 in^2

= **11.25”** (from top)

I = b*h^3/12 + A*d^2 (Parallel Axis)

= (96”*5^3/12) + (96”*5”*(11.25”-2.5”)^2)+ (16”*30^3/12) + (30”*16”*(20”-11.25”)^2) = **110,500 in^4**

S_T = 110,500/11.25” = **9,822.2 in^3**

S_B = 110,500/23.75” = **4,652.6 in^3**

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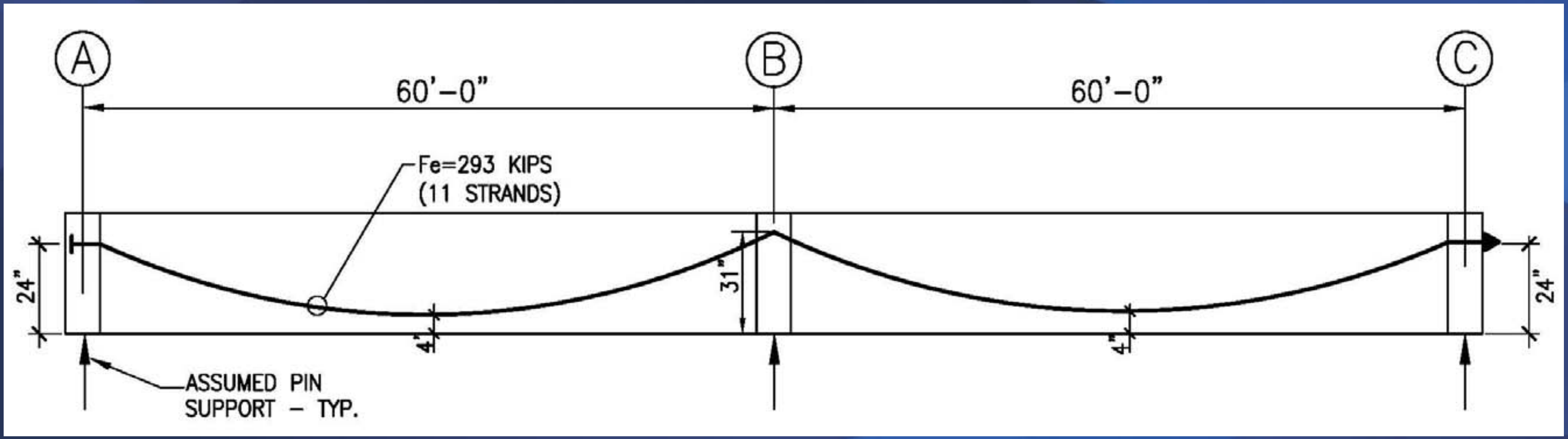
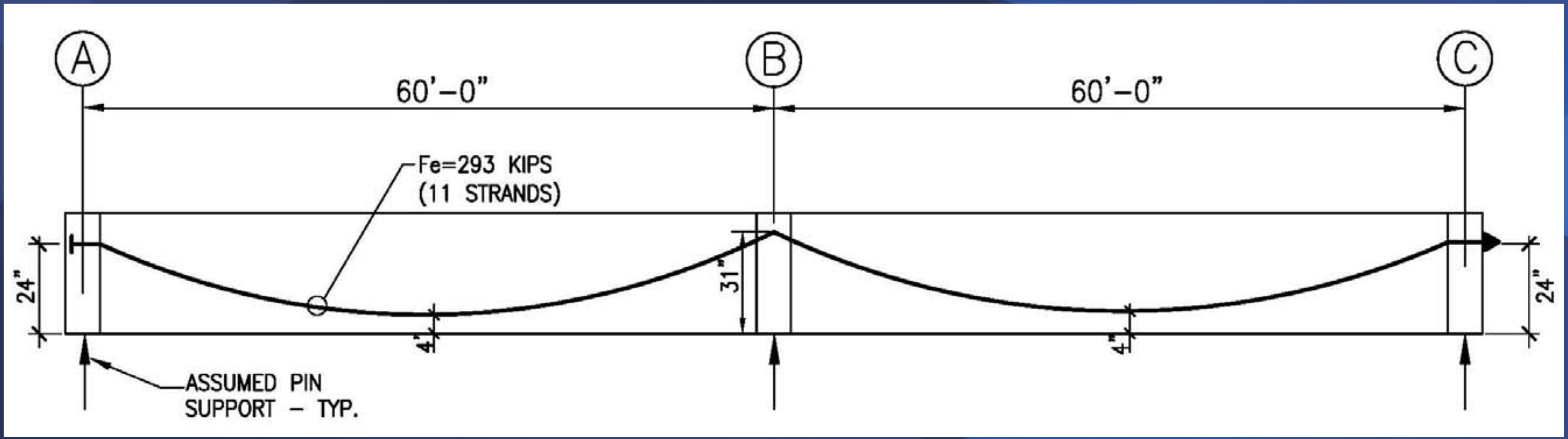
SIMPLIFIED BEAM MODEL



For simplicity of analysis, the exterior columns and interior girder will be assumed to pin/roller support.

Step 1 – Determine balanced loads from the post-tensioning force and its drape.

BALANCED LOAD



Since the tendon is draped (not flat) between the supports, once stressing begins, it will want to "straighten out" to have no curvature between grids A to B and B to C. As it "straightens" it will push upward on the beam.

For a given force, the larger the 'a' dimension, the more upward force is generated as it tries to straighten.

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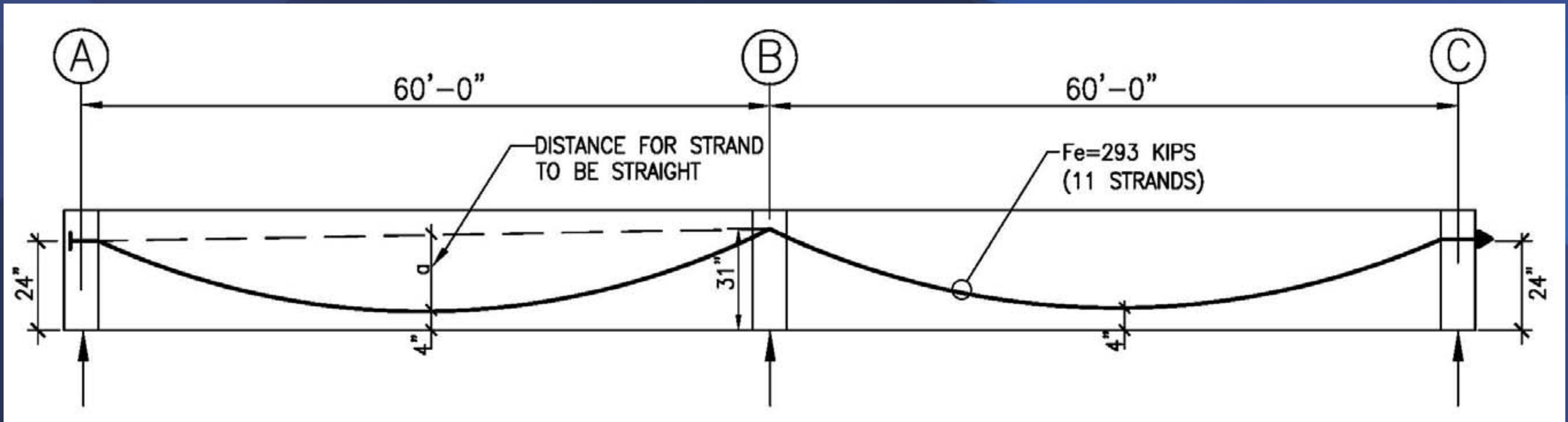
BALANCE LOAD

- The "straightening out" of the strand will push upwards on the concrete section.
- This upward force is called the balance load and is active resistance for the life of the structure.
- Post-tensioning is the **only** reinforcing that pushes back on the structure. This is the primary benefit of post-tensioning and why post-tensioning can save money by using smaller sections and less reinforcing steel. Steel, wood, masonry and metal studs are all passive systems that react to applied load.
- If you don't drape the strands, you are missing the main benefit of post-tensioning!!

CALCULATE THE BALANCE LOAD



$W_{EQ} = (8*F_E*a) / L^2$ a = (31"+24")/2 - 4" = 23.5"

W_{EQ} = (8*293^K*23.5"/12) / 60^2 = **1.28 kips/foot** (each span)

% Conc S.W. = 1.28/1.64 = 0.74 (74%) - The force and profile of the strands removes 74% of the concrete self weight from stress and deflection equations. This is why post-tensioning engineers drape the strands!

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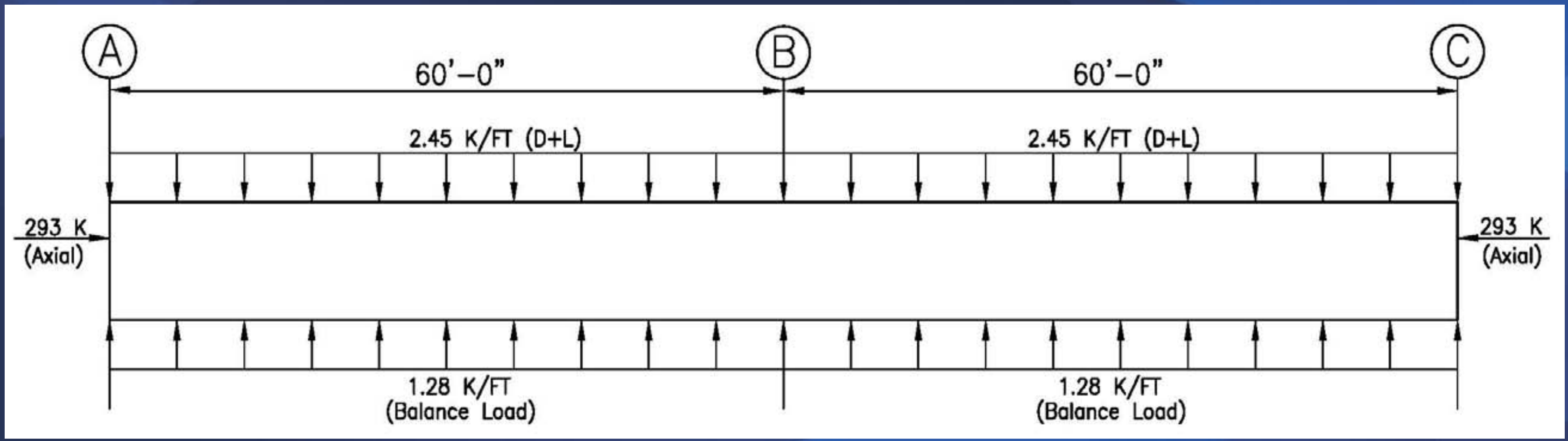
BALANCE LOAD

1. Balance loads are compared to the self weight of the concrete section since only the concrete will be present during stressing.
2. Stressing of the tendons typically occurs 3 to 4 days after placing the concrete so there are no superimposed loads.
3. The percent balanced load is typically between 65 to 100% of the concrete self weight.
4. Our profile is balancing **74%** of the self weight so this layout is in the acceptable range.
5. Balance loads do not need to satisfy any code sections but they are a useful indicator of efficient designs.
6. Having a balanced load significantly greater than 100% of the concrete self weight can lead to cracking, blow outs or upward cambers. Balancing more than 100% should be done with caution.

BEAM MODEL W/ EQUIVALENT LOADS



The beam model shows all loads on the beam. The tendons have been replaced with the load they impart on the beam which is the axial force of the strands and the balance load.

Note: If the balance loads are not opposite of the dead and live load, your drape is wrong and you are not resisting load!

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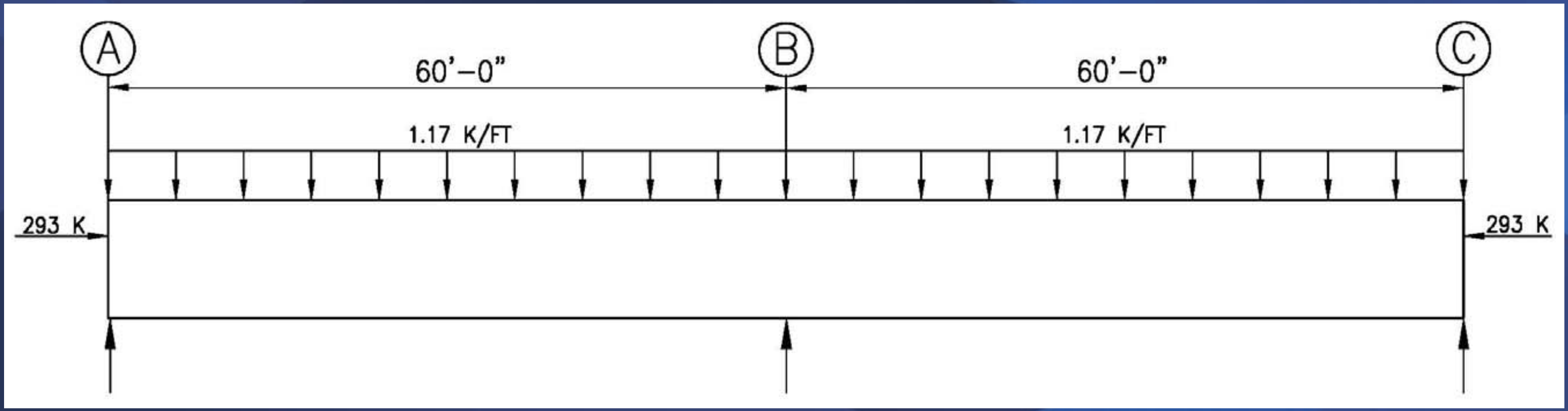
BEAM MODEL W/ NET LOADS



- The net load is generated by subtracting the balance load from the dead and live load.
- The direct force from the anchors is applied at the center of gravity of the section to eliminate any end moments.

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BEAM MODEL W/ NET LOADS

- The net loading will be used to determine the flexural stresses at the critical locations along the span of the beam.
- The net loading is **NOT** used in ultimate strength design. Balance loads are only used to satisfy the allowable stress requirements of the building code.
- The net loading is **NOT** used in determining column or footing loads. Post-Tensioning does not reduce the total weight of the structure.

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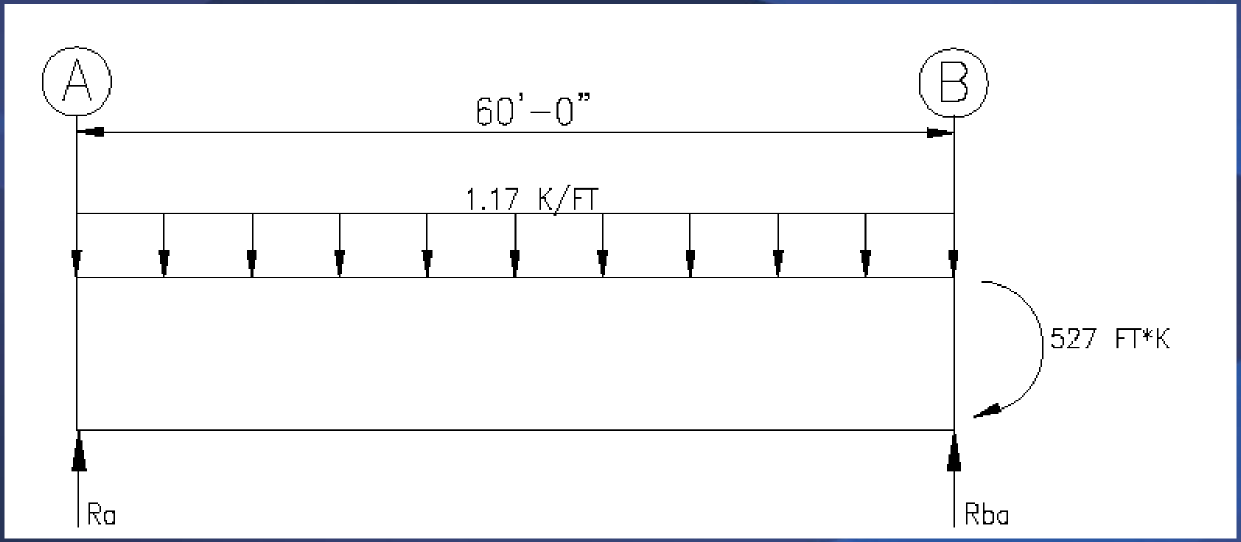
MOMENT AT GRID B

Each span has equal loading and equal spans. With the pin support assumption, the moment at grid B is $W*L^2/8$ per beam theory.

$$M_B = (1.17 * 60^2) / 8 = \mathbf{527\ Ft*Kips}$$

With different spans, support conditions, or loading, an indeterminate structural analysis (moment distribution, computer program, etc.) would be required.

NET SUPPORT REACTIONS



$\sum M_A = -1.17*60^2/8 - 527 + R_{BA}*60' = 0$

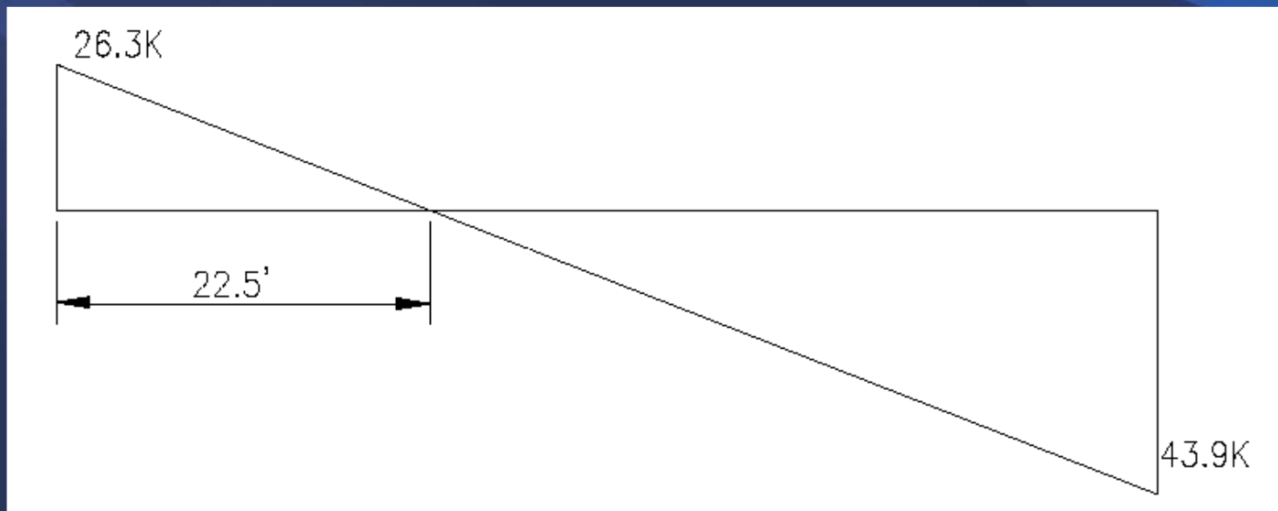
$R_{BA} = 43.9$ Kips (Not to be used in column design)

$\sum M_B = 1.17*60^2/8 - 527 + R_A*60 = 0$

$R_A = 26.3$ Kips (Not to be used in column design)

Check $\sum F_Y$: $1.17*60 = 70.2$ & $43.9+26.3 = 70.2$ **OK**

CALCULATE MID SPAN MOMENT



Net Shear Diagram

$M_{AB} = 26.3^{K} * 22.5' / 2 = 295.9$ Ft*Kips

$M_{BA} = 527^{'K} – 43.9^{K} * (60'-22.5') / 2 = 296.1$ Ft*Kips **OK**

Mid Span Moment = **296 Ft*Kips**

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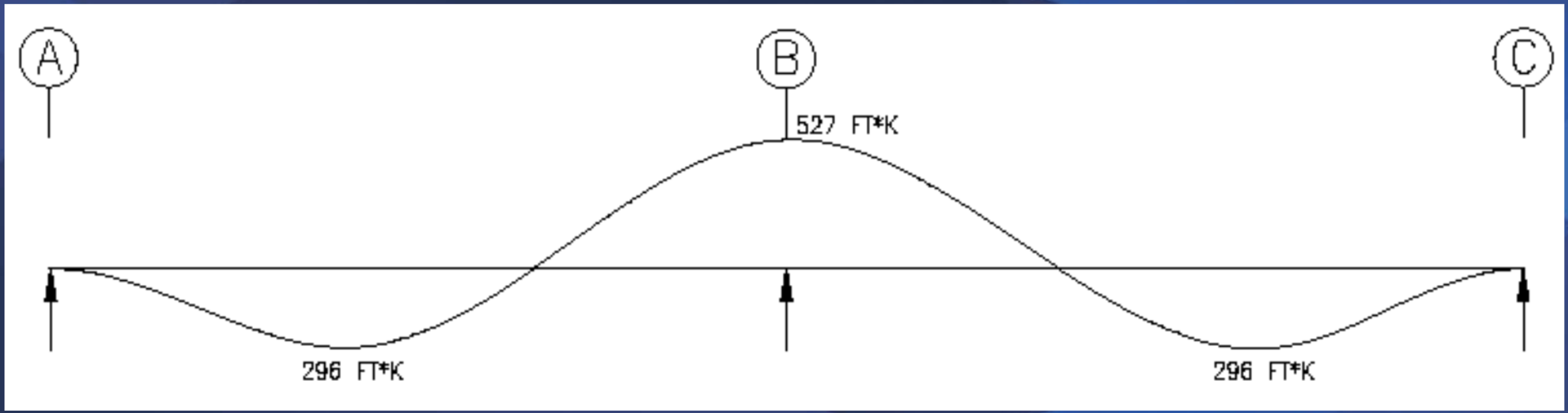
NET SERVICE MOMENT DIAGRAM



- Always draw moment diagrams to the tensile face of the concrete section. The tensile face indicates what portion of the beam requires reinforcing for strength.
- Note the diagram matches the general drape of the tendons. The tendons change their vertical location in the beam to follow the tensile moment diagram. Strands are at the top of the beam over the support and near the bottom at mid span.

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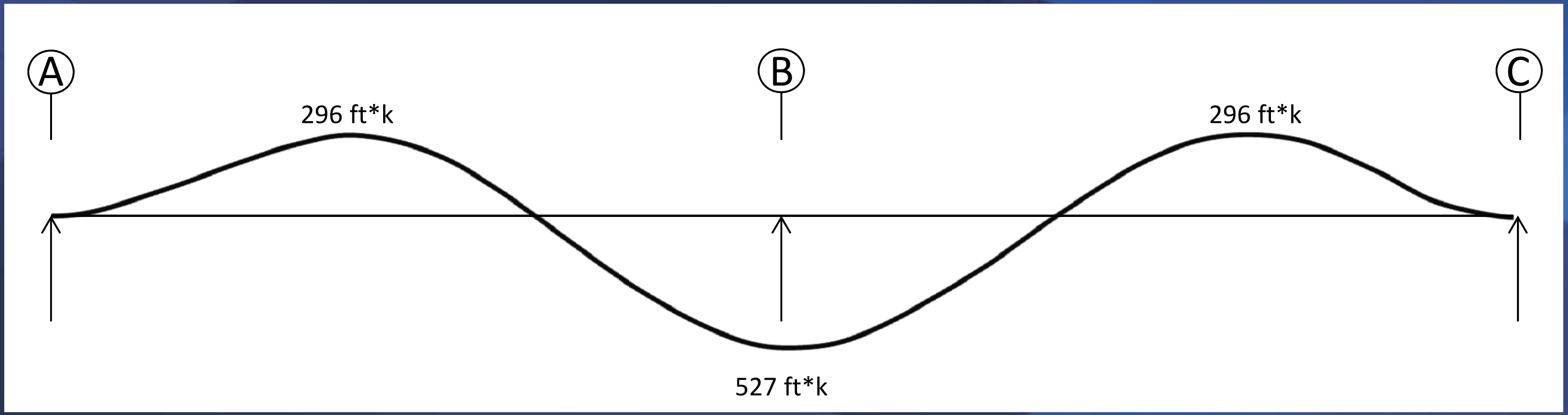
NET SERVICE MOMENT DIAGRAM



(C)

FLEXURAL STRESSES

***Grid B*:** σ_B = P/A +/- M/S

σ_{Btop} = (293 / 960) – (527*12) / 9822.2 = -0.339 ksi (Tension)

σ_{Bbot} = (293 / 960) + (527*12) / 4652.6 = 1.66 ksi (Comp)

***Mid Span*:** σ_{AB} = P/A +/- M/S

σ_{ABbot} = (293 / 960) – (296*12) / 4652.6 = -0.459 ksi (Tension)

σ_{ABtop} = (293 / 960) + (296*12) / 9822.2 = 0.667 ksi (Comp)

WHAT IF WE DIDN'T DRAPE THE TENDONS??

- Without draping the strands, there would be no balance load to offset the dead and live load.

- Only the axial compression would be available to reduce the tensile stresses.

- Placing the strands at the center of gravity of the section would require additional rebar at the locations of high flexural demands.

WHAT IF WE DIDN'T DRAPE THE TENDONS??

With no balance load, the total load is the total dead and live load which is 2.45 kips per foot. With the pin support assumption, the moment at Grid B is $W*L^2/8$ per beam theory.

$M_B = (2.45 * 60^2) / 8 =$ **1,102.50 Ft*Kips**

- Note this would be the same moment if you were designing wood, steel, rebar only concrete, etc.

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WHAT IF WE DIDN'T DRAPE THE TENDONS??

- ***Grid B*:** σ_B = P/A +/- M/S
- σ_{Btop} = (293 / 960) – (1,103*12) / 9822.2 = -1.042 ksi (Tension)
- σ_{Bbot} = (293 / 960) + (1,103*12) / 4652.6 = 3.15 ksi (Comp)
- For no **increase** in cost, draping the strands reduced the flexural stresses from 1.042 ksi to 0.339 ksi which is a reduction of (1.042/.339) 3.07 times.

This is why post-tensioning engineers drape the tendons!!!

SECONDARY MOMENTS

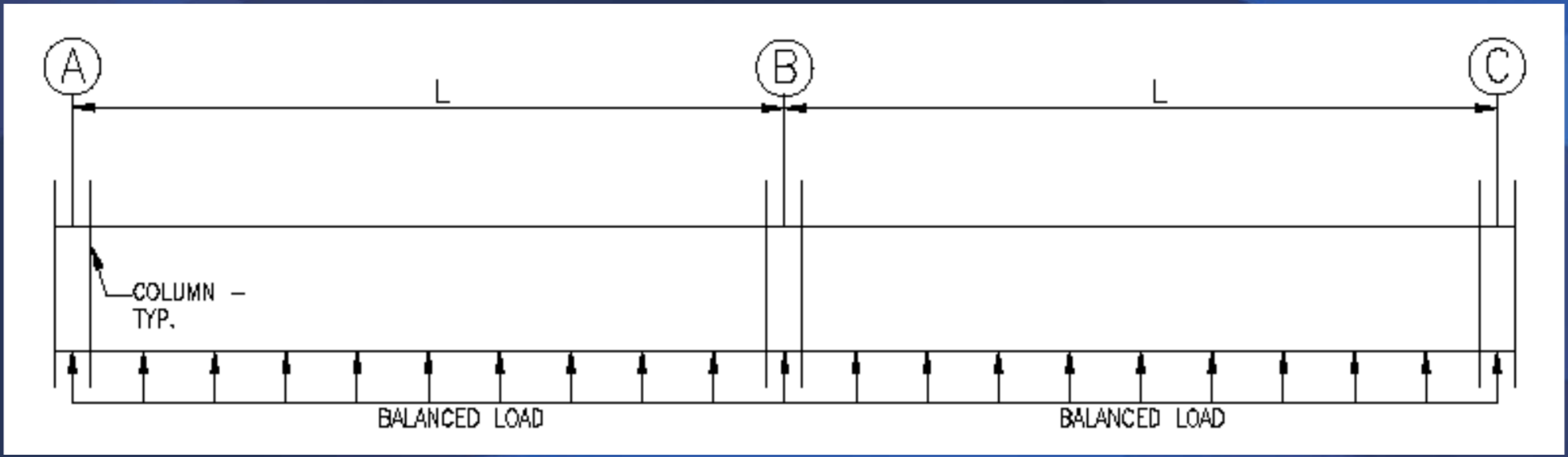
$M_u = 1.2*M_{DL} + 1.6*M_{LL} + 1.0*M_2$ (ACI 18.10.3)

M_{DL} = Dead load Moment M_{LL} = Live Load Moment

M_2 = Secondary Moment caused by draped post-tensioning in an indeterminate system.

Secondary moments do not apply to typical precast sections since the tendons are not draped, there are no uniform balanced loads. In addition, most precast sections are simply supported (determinate) elements which do not create secondary forces. Balanced loads and indeterminacy are required for secondary affects.

SECONDARY MOMENTS – CONCEPT



To explain the concept of secondary moments, lets assume the concrete beam is weightless but stiff enough to restrain the tendons. The tendons are the only force on the section and pushing the beam upward.

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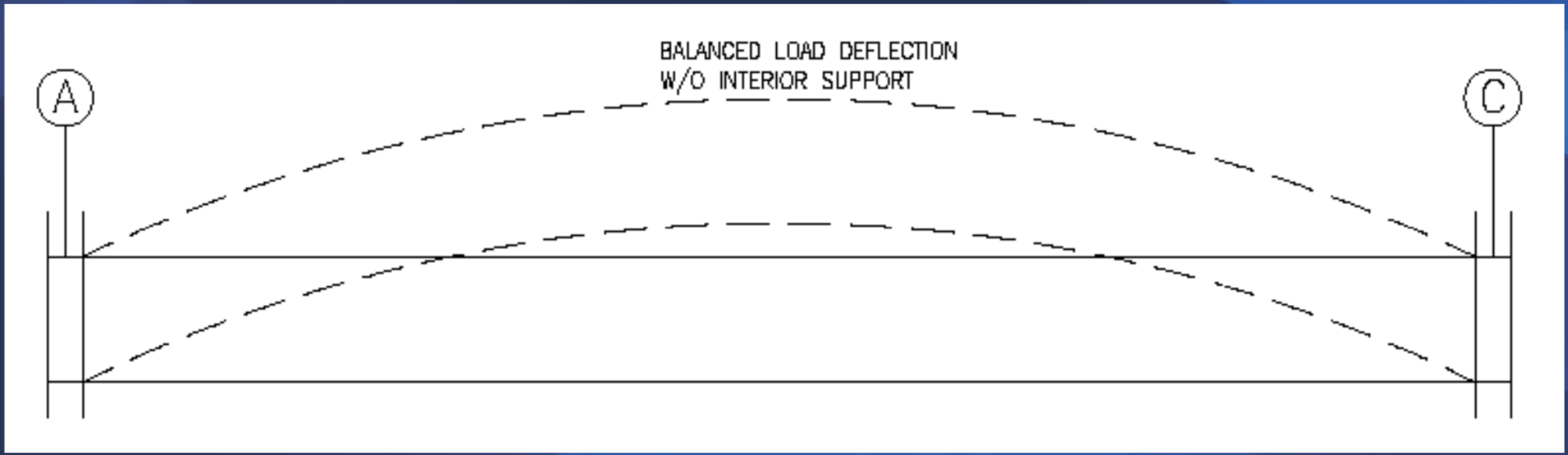
SECONDARY MOMENTS – IDEALIZED DEFLECTION



If we take the interior column (support) away, the uniform balance load will cause the beam to deflected "upward"

In reality, the column and it's foundation restrain this deflection and keep the beam flat at the support.

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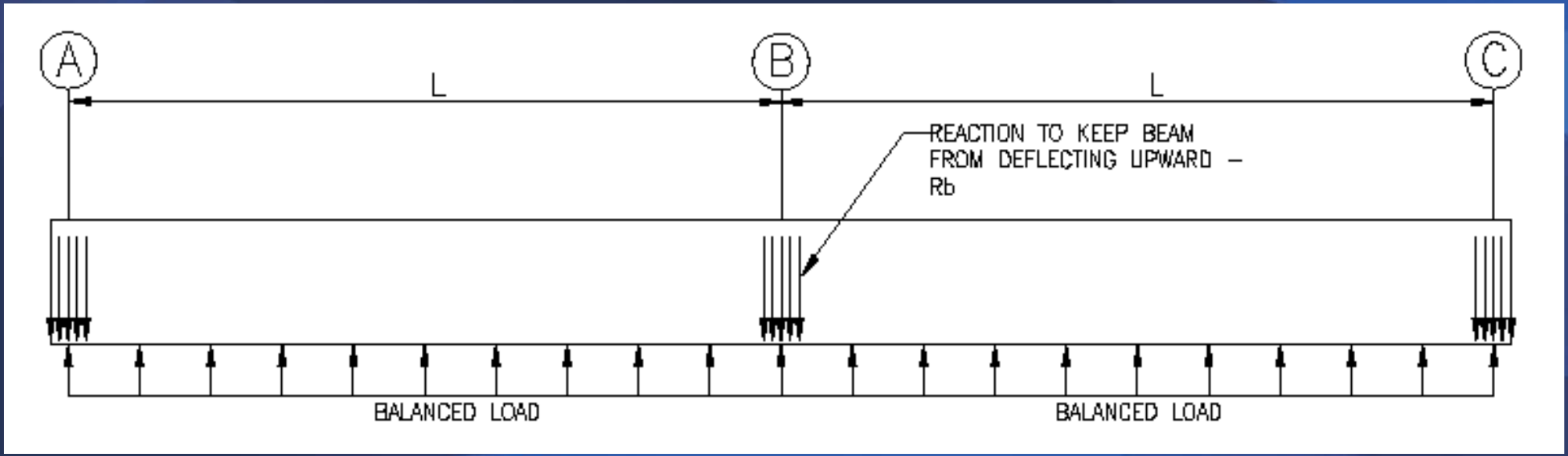
SECONDARY MOMENTS – REACTIONS



To keep the beam deflection zero at grid B, a restraining force is required to counter balance the upward movement.

This can be viewed as a point load on the beam.

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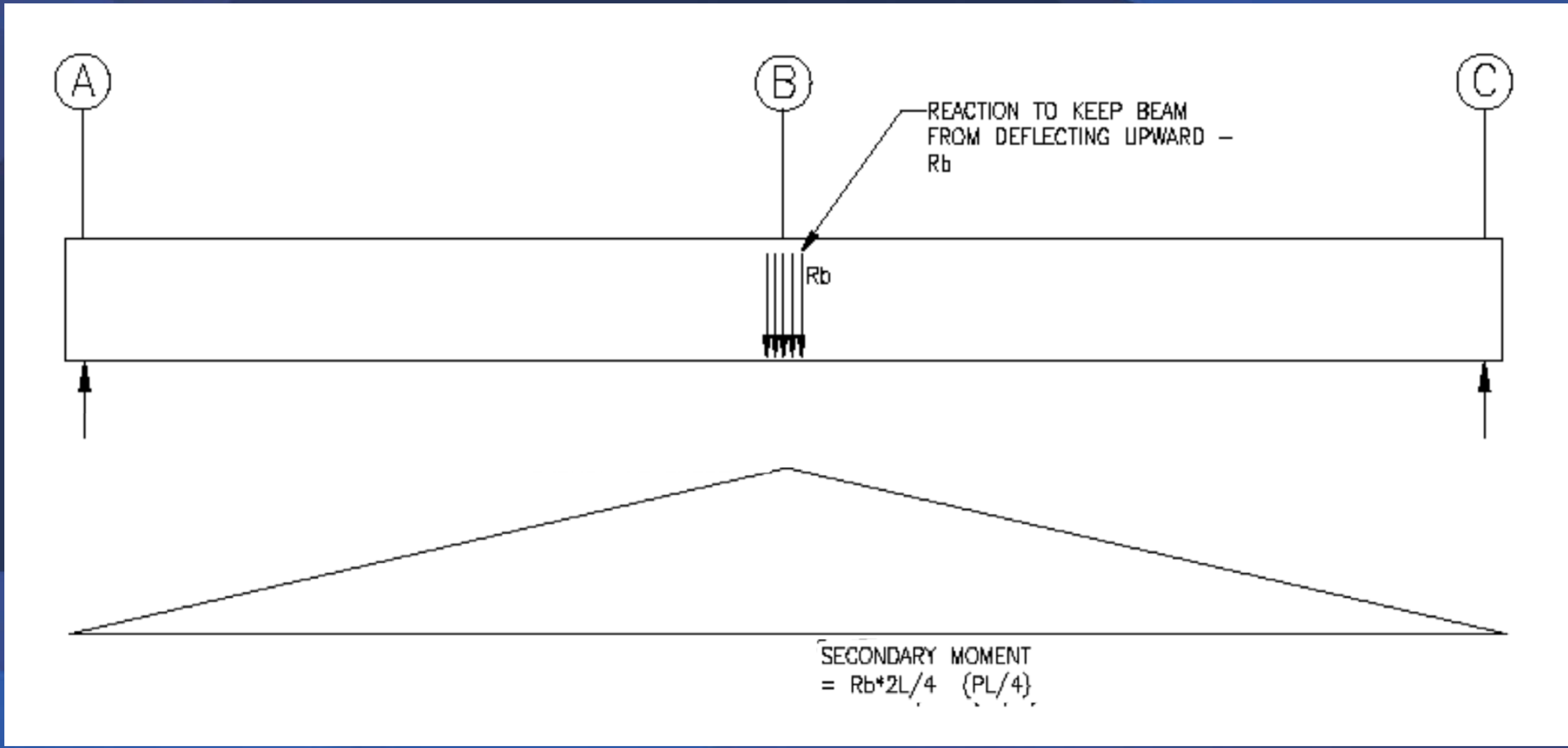
SECONDARY MOMENTS – MOMENT DIAGRAM



With a reaction (concentrated load) replacing the column at mid span, a tension on the bottom moment is generated. Regardless of how many spans, the secondary moment is tension on the bottom for typical slab/beam conditions.

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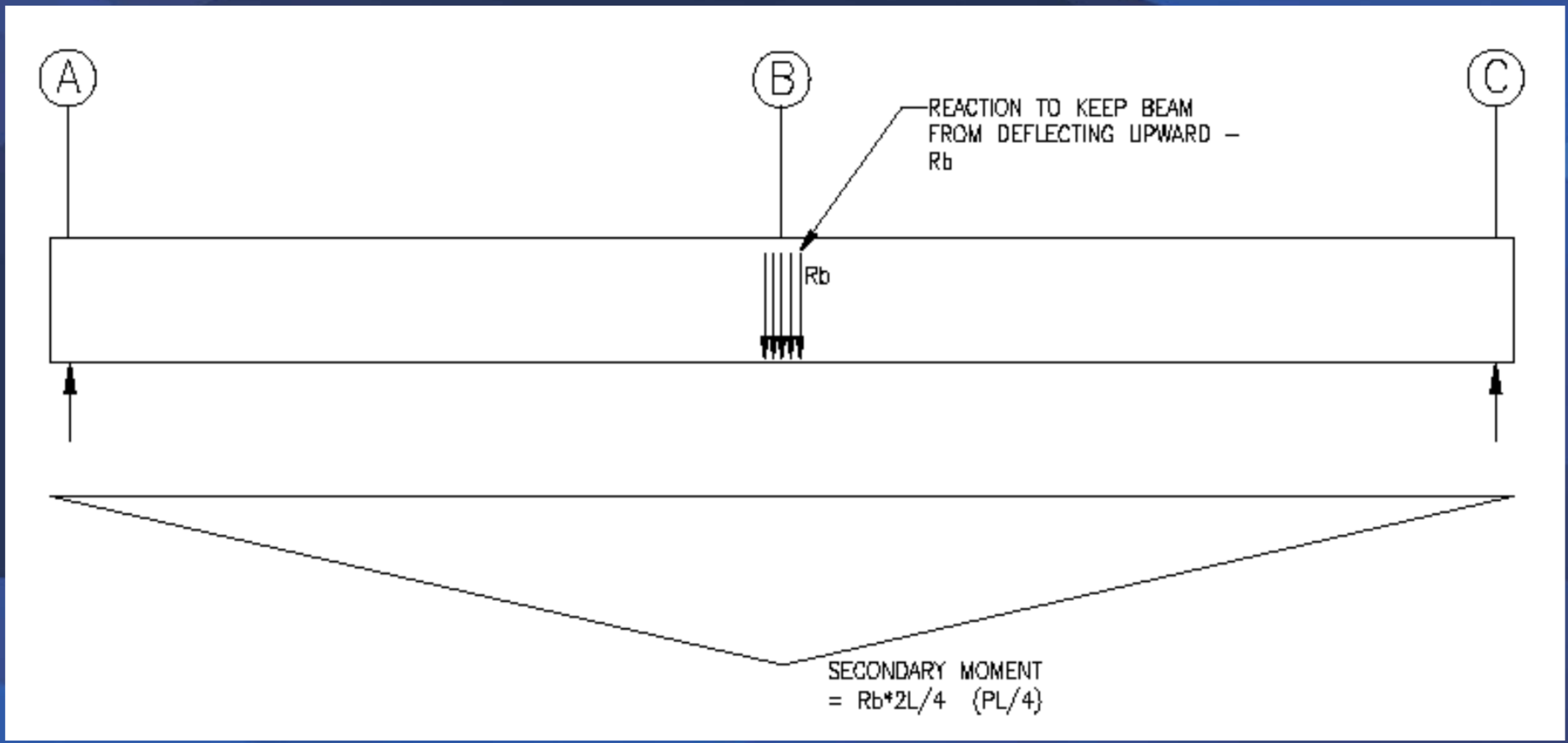
SECONDARY MOMENTS – MOMENT DIAGRAM



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SECONDARY MOMENTS – CONCEPT

- Once the tendons are stressed, their draped profile will cause an upward deflection of the beam.
- The interior support (column, wall or beam) will prevent the beam from deflecting at the support.
- This restraint will create moments in the system that need to be accounted for in the design.
- The 1.0 load factor is used since there will be appreciable no increase in the strand force, therefore no increase the secondary moments.

SECONDARY MOMENTS - CALCULATIONS

The secondary moment is part of the total moment due to the post-tensioning.

$M_{TOTAL} = M_{PRIMARY} + M_2$ → which can be re-written as

$M_2 = M_{TOTAL} - M_{PRIMARY}$

$M_{PRIMARY} = P * Ecc.$ = The tendon force multiplied by the distance between the center of strand (cgs) to the center of the section (cgc). This value will change along the length of the drape. This should be an easy number to calculate.

SECONDARY MOMENTS

In a statically determinate element, $M_{PRIMARY}$ is the M_{TOTAL} since no interior supports exist to create a secondary affect. This is why typical precast members don't have secondary moments.

M_{TOTAL}, $M_{PRIMARY}$ AND M_2

At Grid B: $M_{TOTAL} = 1.28*60^2/8 =$ **576 Ft*Kips**

P = 293 Kips – This is a constant for this beam

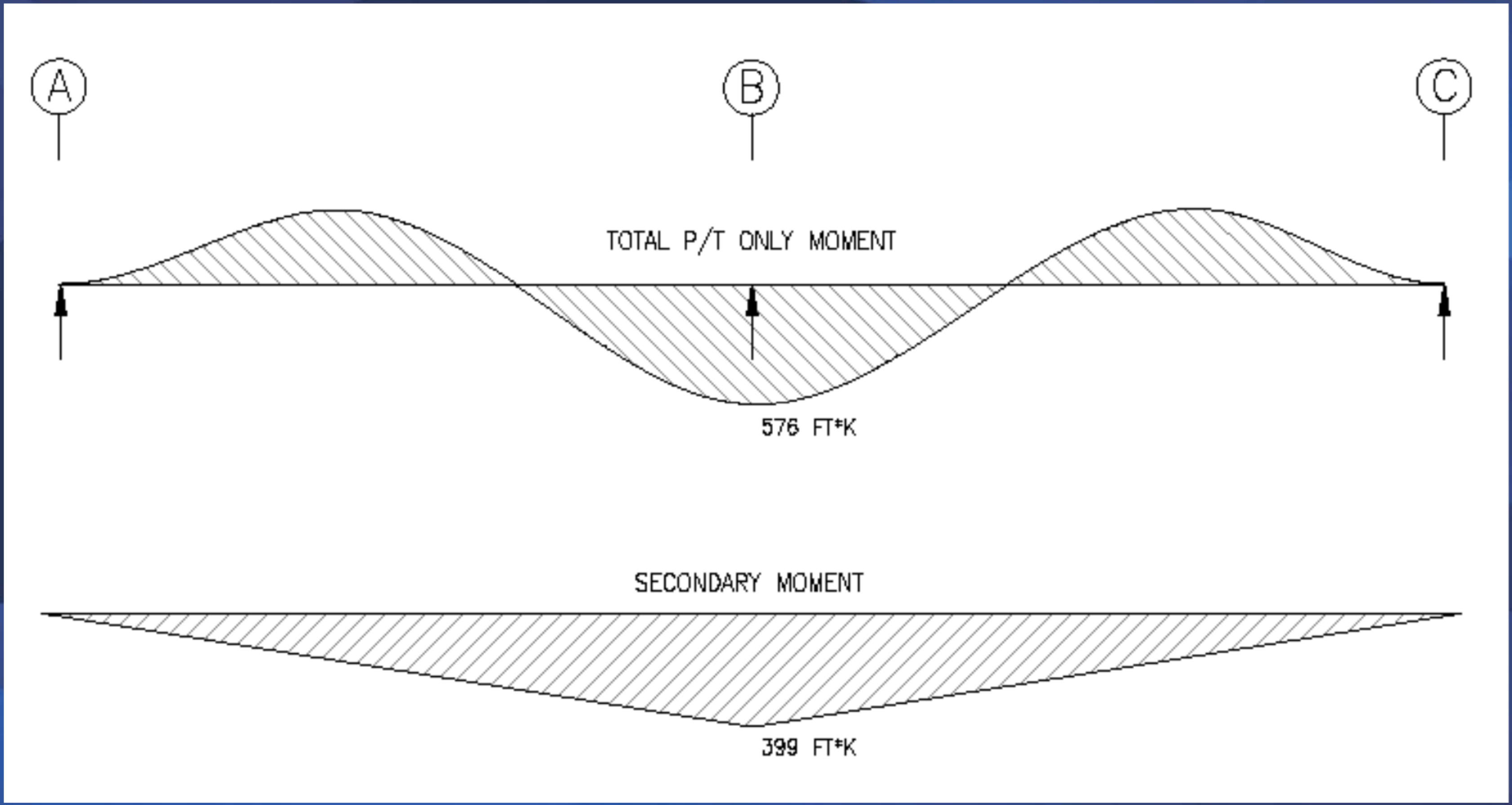
Ecc = 11.25” – 4” = 7.25”

(CGC) (CGS)

$M_{PRIMARY} = 293^{K}*7.25”/12 =$ **177 Ft*Kips**

$M_2 = 576^{Ft\text{-}K} – 177^{Ft\text{-}K} =$ **399 Ft*Kips**

MOMENT DIAGRAMS



Note the M_2 reduces the superimposed (Dead and Live Load) moment at the support while increasing it at mid span. Ignoring M_2 is not conservative.

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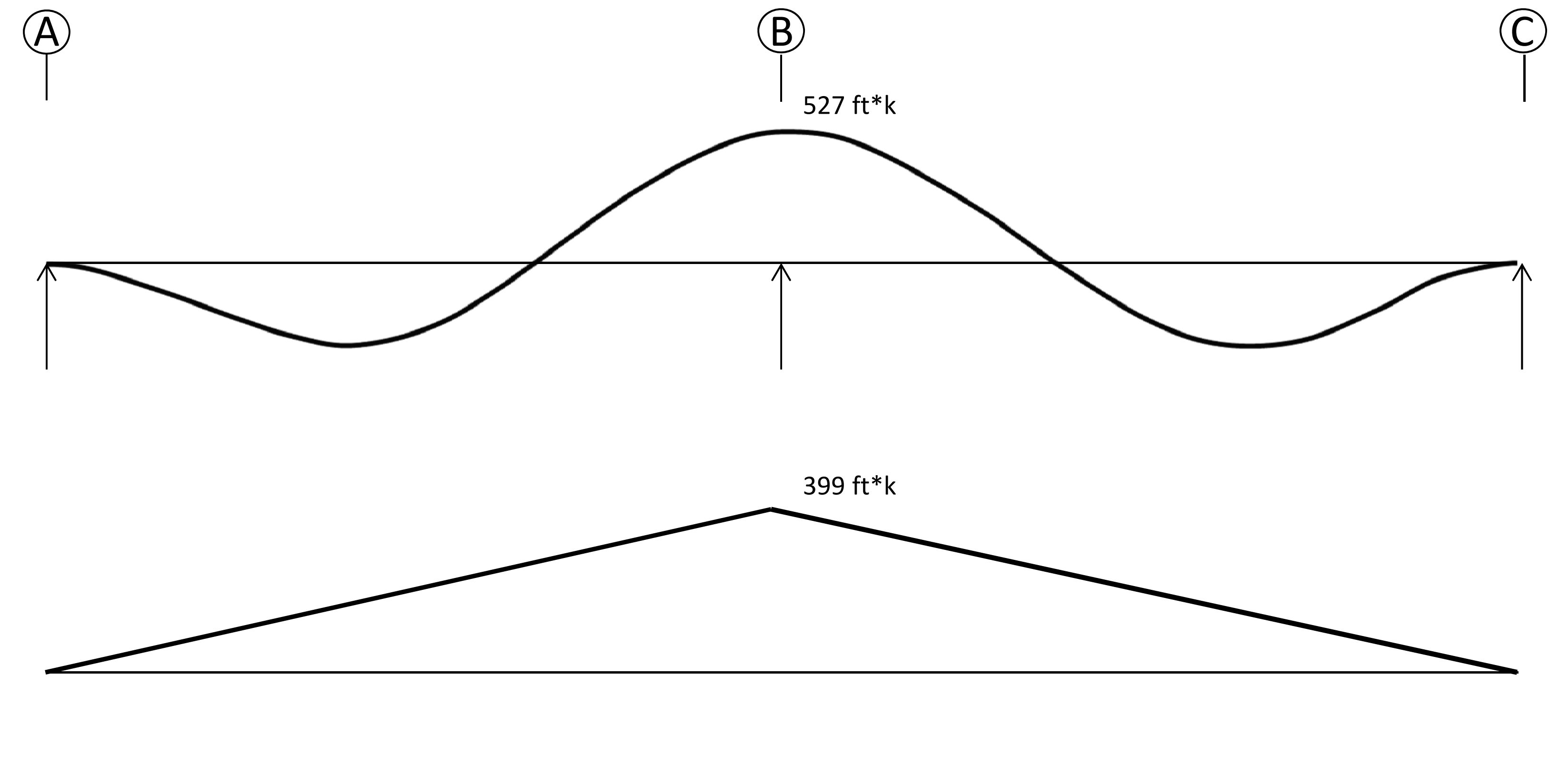
MOMENT DIAGRAMS



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